Sample Paper 6

Leaving Certificate

Mathematics

Paper 1

Higher Level

Time: 2 hours, 30 minutes

300 marks

Examination number

Centre stamp

Running total	

For examiner	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
Total	

Grade

Instructions

There are **two** sections in this examination paper.

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	3 questions

Answer all nine questions.

Write your answers in the spaces provided in this booklet. You may lose marks if you do not do so. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

Marks will be lost if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

Answers should be given in simplest form, where relevant.

Write the make and model of your calculator(s) here:

Section A	Concepts and Skills	150 marks

Answer **all six** questions from this section.

Question 1 **(25 marks)**

(a) A sequence is given by $4T_n = [1+(-1)^n][1+i^n]$, where $i = \sqrt{-1}$. Write out the first four terms and find the sum of the first 100 terms, S_{100}.

(b) Prove by induction that $7^n - 4^n$ is divisible by 3, for all $n \in \mathbb{N}$.

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Question 2 **(25 marks)**

(a) If $z = \cos\left(\frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{3}\right)$ evaluate $|1+z|$, where $i = \sqrt{-1}$.

(b) A is the set of complex numbers such that $|z| = 5$, where $z = x + iy$, where $i = \sqrt{-1}$.

B is the set of complex numbers such that $z + \overline{z} = 8$, where $z = x + iy$, where $i = \sqrt{-1}$.

Plot these sets on an Argand diagram and find $A \cap B$.

Question 3 **(25 marks)**

(a) For a series $S_n = 3\left(1-\left(\frac{1}{3}\right)^n\right)$, find the general term T_n of the series and show it is a geometric series.

(b) 120° is the largest angle in a triangle. $3l$ is the length of the longest side. The lengths of the sides are consecutive terms of an arithmetic sequence. Use the Cosine rule to find the length of each side in terms of l.

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Question 4 (25 marks)

(a) If $x : y = 3:2$ and $x - y = 8$, find x and y.

(b) If you pour half the volume of a small bucket into an empty large bucket, the large bucket is $\frac{3}{8}$ full. Find the ratio of capacity of the large bucket to the small bucket.

(c) Show that $(x+a)(x+b)(x+c) = x^3 + (a+b+c)x^2 + (ab+ac+bc)x + abc$.

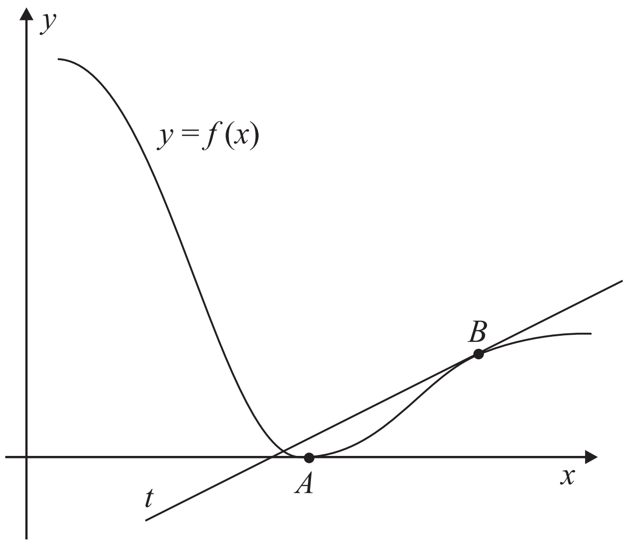
Use this result to write down the answers to:

(i) $(x+1)(x+2)(x+3)$,

(ii) $(x-1)(x-5)(x+7)$.

Question 5 **(25 marks)**

The graph of $y = f(x) = \dfrac{2}{x} - 1 + \ln\left(\dfrac{x}{2}\right)$, $x > 0$ is shown.



(a) Find the coordinates of A, the local minimum of $f(x)$.

(b) Find the coordinates of B, the point of inflection of $f(x)$.

(c) Show that the curve is increasing for all $x > 2$, $x \in \mathbb{R}$.

(d) Show that the equation of t, the tangent at the point of inflection, is given by $x - 8y + 8(\ln 2 - 1) = 0$.

Question 6 **(25 marks)**

(a) Differentiate $y = f(x) = 5 + 4x - x^2$ from first principles.

(b) By completing the square, find the local maximum of $y = f(x) = 5 + 4x - x^2$.

(c) Verify your answer to part **(b)** by differentiating.

(d) Why is $f(x)$ not an injective function?

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Section B Contexts and Applications 150 marks

Answer **all three** questions from this section.

Question 7 **(50 marks)**

Newton's Law of Cooling states that the temperature θ of a body after time t is given by:

$$\theta = T_a + (T_0 - T_a)e^{-kt},$$

where t is the time in hours (h), T_a is the ambient or surrounding temperature, T_o is the temperature at $t = 0$, and k is a positive constant. All temperatures are in degrees Celsius (°C).

(a) If the ambient temperature is 71.6 °F, find its value in °C using the conversion formula $C = \frac{5}{9}(F - 32)$, where F is the temperature in °F and C is the temperature in °C.

(b) If $k = 1.8$ h^{-1}, find the temperature of a cup of tea, to the nearest degree, after half an hour, given that its initial temperature was 96°C and the ambient temperature was 22°C.

(c) Rearrange the formula $\theta = T_a + (T_0 - T_a)e^{-kt}$ to show that $kt = \ln\left(\frac{T_0 - T_a}{\theta - T_a}\right)$.

(d) The state pathologist visits the scene of a suspected murder. She arrives at 10:30 p.m. and begins to take some measurements. The temperature of the body was 27ºC. The thermostat shows that the room has been kept at a constant temperature of 20ºC for the past week. The temperature of the body is taken one hour later and is found to be 26ºC.

(i) Assuming the victim's body temperature was the normal 37ºC prior to his death, complete the following table, in terms of t:

t (hours)	θ (ºC)
	37
t	27
	26

T_a = ____________________

T_0 = ____________________

(ii) Using Newton's Law of Cooling, find k, to three decimal places.

(iii) Estimate the time of death, to the nearest minute.

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Question 8 **(50 marks)**

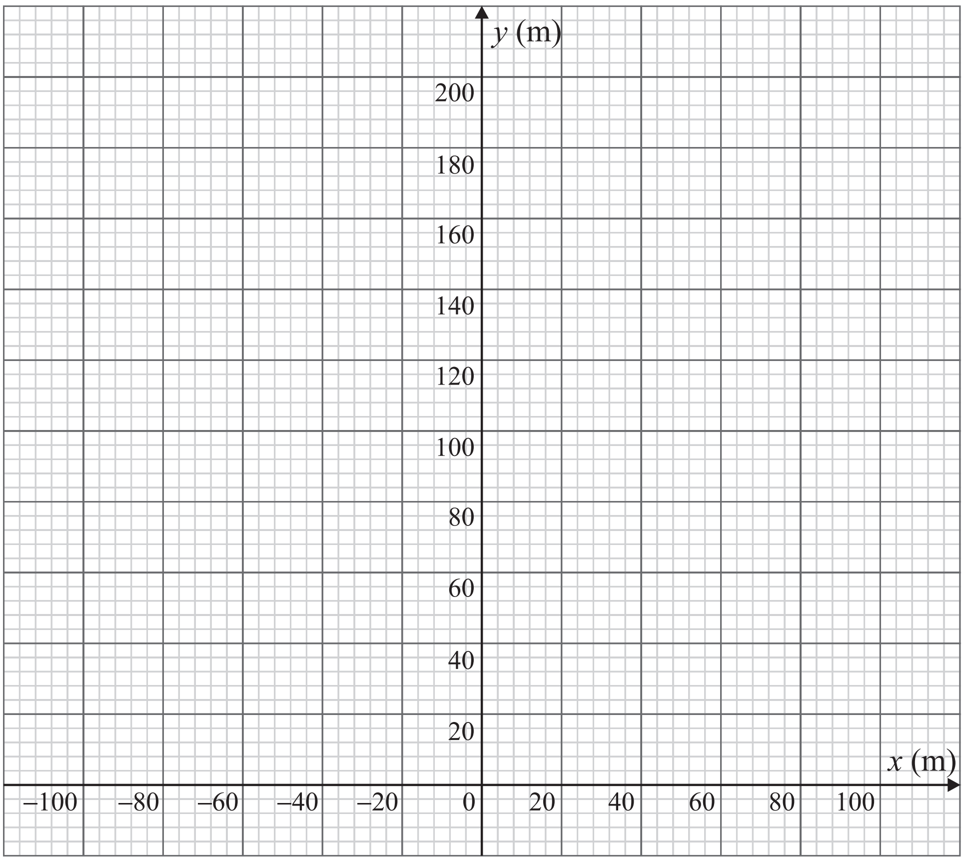
The St Louis Gateway Arch is often mistakenly described as an inverted catenary arch. A catenary is the curve formed when you allow a chain or cable to hang from its two ends under its own weight. Therefore, a catenary is the curve of a hanging chain.

(a) The St Louis Gateway Arch is described by the equation $y = 231 - 19.57\{e^{\frac{1}{39}x} + e^{-\frac{1}{39}x}\}$ for $-96 \le x \le 96$, where x and y are measured in metres (m).

(i) Fill in the following table rounding off y to the nearest metre:

x (m)	–96	–64	–32	0	32	64	96
y (m)							

(ii) Plot a graph on the grid below.



(b) **(i)** Use your graph to find the maximum height of the St Louis Gateway Arch and its height when $x = 50$ m.

(ii) The true value of its maximum height is 630 feet. Find the percentage error in your model, to three significant figures. [1 foot = 0.3048 m]

(c) Use the trapezoid rule to find the area under the Gateway Arch, to the nearest square metre.

(d) **(i)** e^x can be approximated as $e^x \simeq 1 + x + \frac{1}{2}x^2$. Show that $e^x + e^{-x} \simeq 2 + x^2$.

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(ii) Hence, show that $y = 231 - 19.57\{e^{\frac{1}{39}x} + e^{-\frac{1}{39}x}\}$ can be approximated to $y \simeq 191.86 - 0.013x^2$.

(e) The base width of the Gateway Arch is 192 m. Find what width, to the nearest whole number, is given by the approximate model $y \simeq 191.86 - 0.013x^2$.

(f) To what value would you have to change the coefficient of x^2 in $y \simeq 191.86 - 0.013x^2$ so that the base width is 192 m, to four decimal places?

Question 9 **(50 marks)**

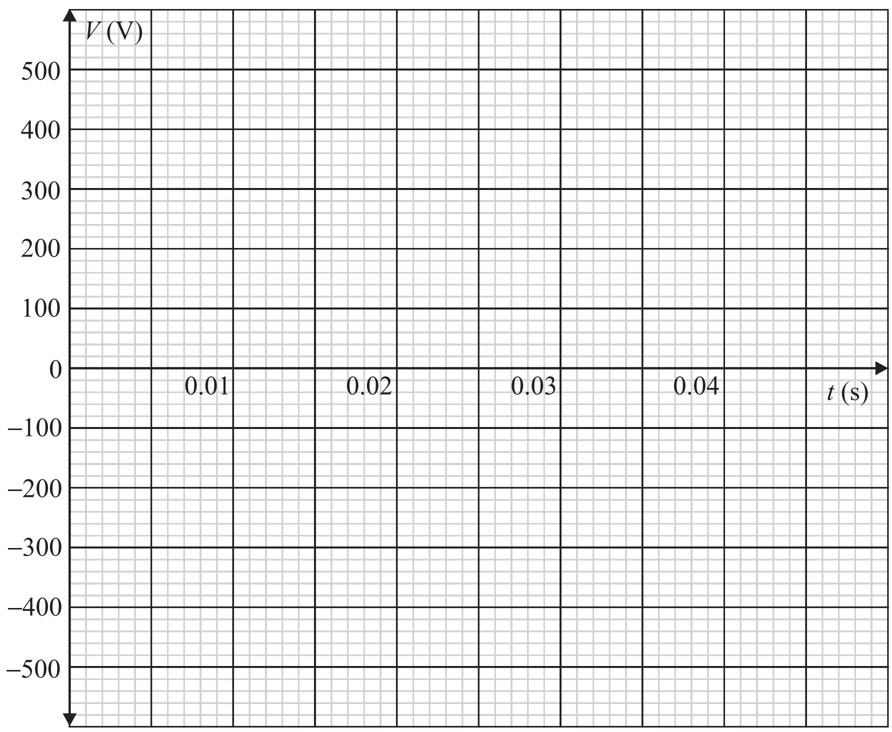
The ESB supplies an alternating voltage according to the formula $V = 340\sin(100\pi t)$, where V is the voltage in Volts (V) and t is the time in seconds (s).

(a) Find the period and range of this voltage.

(b) Find the maximum voltage (known as the peak voltage).

(c) Plot this voltage for values $0 \text{ s} \leq t \leq 0{\cdot}04 \text{ s}$.

t	0	0·005	0·01	0·015	0·02	0·025	0·03	0·035	0·04
$100\pi t$									
$\sin(100\pi t)$									
$340\sin(100\pi t)$									



(d) Show that the average value of V over a period is 0 V.

(e) Find $\dfrac{dV}{dt}$ **(i)** at $t = 0$ s, **(ii)** at $t = 0.005$ s, **(iii)** at $t = 0.01$ s.

(f) Show that $V^2 = 57800(1-\cos(200\pi t))$. Find the period of V^2. Find the average value of V^2 over a period.